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Short Communication

Polynomial approach for calculating added mass for fluid-filled cylindrical shells

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Abstract

Cylindrical shells find wide applications in many engineering fields. Free vibration analysis of cylindrical shells filled with fluid has been dealt using finite element approach for both structure and fluid domain or using finite element for structure and Bessel function approach for fluid. The present paper deals with a novel method on the usage of polynomial functions for fluid domain in contrast to the usual Bessel function approach. A semi-analytical finite element approach has been used to discretise the shell structure. The fluid domain has been analysed by using polynomial functions instead of Bessel function. The study has been carried out for conventional shells as well as viscoelastic shells. The present study obviates necessity of limiting the studies to certain boundary conditions. The results of both frequency and damping corroborate well with those found in literature.

1. Introduction

Frequency analysis of fluid-filled cylindrical shells has been of great interest and a challenging task. In literature, fluid-filled cylindrical shells have been analysed for their vibratory behaviour either by using finite element or by boundary solution technique. The vibration behaviour of conventional plates and shells have been analysed widely by using the concept of polynomial or trigonometric functions for displacement satisfying the appropriate boundary conditions. Haroun

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[1] has carried out earthquake analysis by using Bessel function approach for fluid domain and finite element method for structure. However, his studies were limited to axial and first circumferential modes. Ramasamy and Ganesan [2] have carried out studies on fluid-filled viscoelastic shells by using semi-analytical method. Amabili [3] has studied fluid-filled shells by experimental results and along with closed-form solutions. In his method, both structure and fluid domain are treated by using boundary solution technique. Such an approach needs to identify set of trail functions, which satifies wave equation as well as boundary conditions. Amabili [4] studied the free flexural vibrations of a partially fluid-loaded simply supported circular cylindrical shell for various wet angles. The fluid is assumed to be inviscid along with a free surface parallel to the shell axis. The presence of either external or internal fluid is studied for both compressible and incompressible cases using the virtual added mass approach. The present studies are of great importance considering the fact that the literature on damping characteristics of horizontal fluid-filled cylindrical shells is sparse and studies are limited to conventional cylindrical shells.

In the present study instead of using the above-said methodologies, a polynomial is chosen to represent the variation of pressure along the radial direction. The fluid stiffness matrix and interaction matrix are evaluated from which the added mass of the system is deduced. From literature it is found that such an approach for fluid domain has not been attempted for cylindrical shells.

2. Structural finite element formulation for conventional shell

In the present paper, cylindrical shells are analysed using semi-analytical finite element approach and polynomial function approach for fluid domain. The following sections describes the methodology.

2.1. Three-noded axisymmetric shell element

A typical finite element discretisation of mid-surface of the cylindrical shell using 3-node quadratic line elements is shown in Fig. 1. In the present study for structral problem finite element developed by Ramalingeswarao and Ganesan [5] is made use of.

The element is based on first-order shear deformation theory. It is assumed that the normal to the reference surface before deformation remains the same even after deformation. The normal strain is neglected. The thickness is assumed to be small compared to its radius of curvature of the shell and the normal does not undergo any strain.

Fig. 2 shows a schematic of the 3-noded axisymmetric shell element.

Each node of the element has 5-degrees of freedom, viz., u, v, w, ψ_s, ψ_θ. The displacement field for the shell is expressed as

$$U(s,\theta,z) = u + z\psi_s, \quad V(s,\theta,z) = v + z\psi_\theta, \quad W(s,\theta,z) = w(s,\theta). \tag{1}$$

The stresses in the element are

$$\{\sigma\} = \{\sigma_{ss} \quad \sigma_{\theta\theta} \quad \tau_{\theta z} \quad \tau_{sz} \quad \tau_{s\theta}\}^{\mathrm{T}} \tag{2}$$

and the corresponding strain–displacement are given by Rao and Ganesan [5].

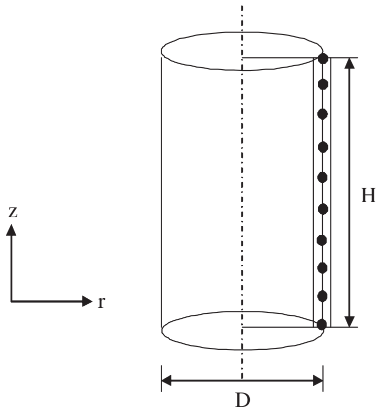

Fig. 1. Finite element descretisation of cylindrical shell.

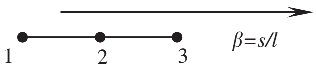

Fig. 2. Three-noded axisymmetric shell element.

The shape functions for the 3-noded shell element are as follows:

$$\psi_1 = \tfrac{1}{2}(\xi^2 - \xi), \quad \psi_2 = (1 - \xi^2), \quad \psi_3 = \tfrac{1}{2}(\xi^2 + \xi). \tag{3}$$

In the semi-analytical approach, the displacements and rotations of any point in the element are expressed in terms of the nodal displacements and rotations as follows:

$$u = \sum_{n=0}^{\infty} u_n \cos n\theta, \quad v = \sum_{n=0}^{\infty} v_n \sin n\theta, \quad w = \sum_{n=0}^{\infty} w_n \cos n\theta,$$
$$\alpha = \sum_{n=0}^{\infty} \alpha_n \cos n\theta, \quad \beta = \sum_{n=0}^{\infty} \beta_n \sin n\theta, \tag{4}$$

where 'n' denotes the circumferential mode number.

For the nth mode the mass and stiffness matrices are given by

$$[k_e] = \int_{\text{Area}} [B]^{\mathrm{T}}[D][B] r \, \mathrm{d}\theta \, \mathrm{d}s, \quad [m_e] = \rho \int_{\text{Vol}} [\psi]^{\mathrm{T}}[\psi] r \, \mathrm{d}\theta \, \mathrm{d}r \, \mathrm{d}s, \tag{5}$$

where $[B]$ is the strain-displacement matrix, $[D]$ is the stress–strain matrix with thickness taken into account, $\{\psi\}$ the matrix of shape function.

3. Finite element formulation of constrained layer shell element

Ramasamy and Ganesan [2] have developed a general shell finite element for viscoelastic shells based on the displacement field proposed by Wilkins et al. [6]. Fig. 3 shows the schematic of the

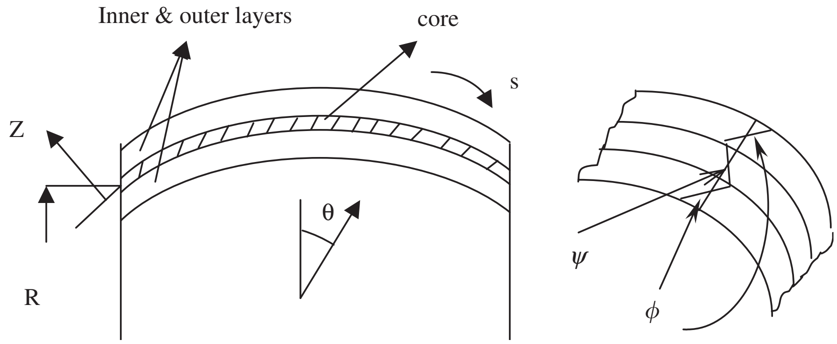

Fig. 3. Schematics of a constrained viscoelastic layer.

viscoelastic shell structure, consisting of a core viscoelastic layer sandwiched between two facing layers. The extreme layers of the shell are called facings, and their individual thicknesses are denoted by t_f. The central portion of the shell is constrained viscoelastic layer which is called core and the thickness of the same is denoted by t_c.

For the core layer the displacement relations are assumed to be

$$u^c = u_o + z\psi_s, \quad v^c = v_o + z\psi_\theta, \quad w^c = w_o, \tag{6}$$

where u^c, v^c and w^c are the total displacements in the s-, θ-, and z-directions and are defined in terms of the middle surface displacements u_o, v_o and w_o and the angles, ψ_s and ψ_θ, are rotations of normal to the middle surface in the meridional and circumferential directions. For the core these angles are denoted by ψ_s, ψ_θ and for the facings the angles are denoted as ϕ_s and ϕ_θ.

The displacement relations for outer and inner facing are, respectively,

$$u^{fo}, u^{fi} = u_o \pm h\psi_s + (z \pm h)\phi_s; \quad v^{fo}, v^{fi} = v_o \pm h\psi_\theta + (z \pm h)\phi_\theta; \quad w^{fo}, w^{fi} = w_o. \tag{7}$$

Here 'z' denotes the distance from the middle surface of the shell, 'h' is half the core thickness and R is the radius of the shell with respect to the axis. The strain–displacement relations for the core, inner and outer facings are given by Ramasamy and Ganesan [2].

4. Fluid formulation

Wave equation for incompressible fluid in terms of pressure is given by

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} = 0. \tag{8}$$

In general, the series solution can be adapted to solve this equation. From wave equation, the variation form can be deduced and the following equations for stiffness and interaction matrix can be deduced.

The variational form can be used to derive the elemental fluid stiffness matrix.

$$[h_e] = \int_{\text{vol}} \left(\frac{\partial[\bar{N}]^{\text{T}}}{\partial r} \frac{\partial[\bar{N}]}{\partial r} + \frac{1}{r^2} \frac{\partial[\bar{N}]^{\text{T}}}{\partial \theta} \frac{\partial[\bar{N}]}{\partial \theta} + \frac{\partial[\bar{N}]^{\text{T}}}{\partial z} \frac{\partial[\bar{N}]}{\partial z} \right) r \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}z,$$

where N is the shape function of the fluid domain.

The liquid adjacent to the wall of the elastic shell, $r = R$ must move radially with the same velocity as the shell

$$\frac{\partial p}{\partial r}(R, \theta, z, t) = -\rho \frac{\partial^2 w}{\partial t^2}(\theta, z, t), \tag{9}$$

where $w(\theta, z, t)$ is the shell radial displacement and ρ is density of liquid.

By using the variational form of Eq. (9) the following interaction matix is derived:

$$[s_e] = \int_A [\bar{N}]^{\text{T}} [N_{\text{str}}] \, d(A_{\text{inter}}), \tag{10}$$

where A_{inter} represents the area of interaction and $[N_{\text{str}}]$ corresponds to shape function pertaining to normal to shell displacement.

Once a set of trail functions, $\bar{N}(r, \theta, z)$ which satisfy the boundary condition, are identified, it is possible to evaluate the above matrices and get stiffness and interaction matrices. Once elemental stiffness and interaction matrices are calculated, they will be assembled to evaluate the fluid stiffness matrix $[H]$ and fluid interaction matrix $[S]$ and added mass matrix is calculated. In the present study, trail functions chosen in the z-direction as well as in θ-direction are same as that used in the Bessel function approach.

$$\bar{N}(r, \theta, z) = \sum_{i}^{I} \sum_{j=1}^{4} A_{ji}.F_j(r) \, \cos(\alpha_i z) \, \cos(n\theta), \tag{11}$$

where

$$\alpha_i = \frac{(2i-1)\pi}{2H} \quad \text{and} \quad i = 1, 2 \ldots I.$$

The following boundary conditions are to be satisfied while choosing $F_j(r)$:

$$p = 0 \text{ when } r = 0, \quad p \neq 0 \text{ when } r = R, \quad \frac{\partial p}{\partial r} \neq 0 \text{ when } r = R. \tag{12}$$

The series $(r/a)^n$ will satisfy the above boundary condition for any n. In the present study, $n = 1$, 2, 3 and 4 polynomial function have been chosen to represent $F_j(r)$. In addition, 10 axial modes have been made use of in present study.

A computer program is developed to find stiffness matrix of the fluid and the interaction matrix of the fluid. The matrices will become decoupled in θ-direction.

After calculating stiffness and interaction matrices added mass is calculated and added to structral mass matrix to find eigenvalues of fluid-filled cylindrical shell.

The added mass is calculated by the following equation:

$$[M_a] = [S]^{\text{T}} [H]^{-1} [S], \tag{13}$$

where $[H]$, $[S]$ are global fluid stiffness and interaction matrices.

$$([M_s] + [M_a])\{\ddot{q}\} + [K_s]\,\{q\} = \{0\}, \tag{14}$$

where ADM is an added mass matrix due to the effect of the liquid. The matrix [ADM] is symmetric and partially complete. Eq. (14) can be reduced to the standard eigenvalue problem as shown below which will be solved for evaluating eigenvalues.

$$\left\{[K_S] - [[M_S] + [ADM]]\right\} = 0. \tag{15}$$

5. Results and discussions

A computer programme developed to evaluate the natural frequencies of conventional shells by using polynomial function has been used to study short and tall shells and that are dealt by Amabili [3]. In the present study three shells whose dimensions are $R = 18.29\,\text{m}$, $L = 12.2\,\text{m}$,

Table 1
Comparison of natural frequencies (Hz) for a clamped–free, mild steel shell of dimensions $R = 18.29\,\text{m}$, $L = 12.2\,\text{m}$, $t = 0.0254\,\text{m}$

Circumferential mode number	Ramasamy and Ganesan [2]	Polynomial approach
1	6.34	6.28
2	5.27	5.25
3	4.19	4.19
4	3.35	3.36
5	2.72	2.73
6	2.24	2.26
7	1.88	1.90
8	1.62	1.65
9	1.45	1.48
10	1.35	1.40

Table 2
Comparison of natural frequencies (Hz) for a clamped–free, mild steel shell of dimensions $R = 7.34\,\text{m}$, $L = 21.96\,\text{m}$, $t = 0.0254\,\text{m}$

Circumferential mode number	Ramasamy and Ganesan [2]	Polynomial approach
1	5.41	5.35
2	2.51	2.47
3	1.45	1.43
4	1.10	1.10
5	1.25	1.24
6	1.72	1.72
7	2.42	2.42
8	3.28	3.29
9	4.30	4.33
10	5.49	5.56

$t = 0.0254$ m (short shell), $R = 7.32$ m, $L = 21.96$ m, $t = 0.0254$ m (long shell) and $R = 0.175$ m, $L = 0.664$ m, $t = 0.001$ m (shell dimension used by Amabili [3]) considered. Tables 1–3 compare the results obtained by polynomial approach and semi-analytical method suggested by Ramasamy and Ganesan [2]. It is observed that there is good correlation.

In order to further validate the polynomial function approach the study has been carried out on constrained layered composite shell. The study has been carried out for fibre orientation 0° and 90°. Tables 4 and 5 compare the frequencies and damping values obtained by polynomial approach with those of Ramasamy [2]. It is seen from tables that there is a good correlation on the frequencies and damping values obtained by both the approaches.

Table 3
Comparison of natural frequencies (Hz) for a simply supported, mild steel shell of dimensions $R = 0.175$ m, $L = 0.664$ m, $t = 0.001$ m

Circumferential mode number	Polynomial approach	Amabili [7] experiments	Amabili [7] theory
1	89.96	92	91.0
2	101.10	104	102.8
3	116.70	119	117.2
4	139.60	147	141
5	196.50	206	197
6	197.60		
7	268.60		
8	355.60		
9	405.50		
10	457.60		

Table 4
Comparison of natural frequencies (Hz) for a clamped–free, Glass for Epoxy shell with fibre angle 0° of dimensions $R = 18.29$ m, $L = 12.2$ m, $t = 0.0254$ m

Circumferential mode number	Polynomial approach		Ramasamy and Ganesan [2]	
	Frequency (Hz)	Loss factor	Frequency (Hz)	Loss factor
	$t_c/t_f = 1$	$t_c/t_f = 1$	$t_c/t_f = 1$	$t_c/t_f = 1$
1	1.34	0.0034	1.34	0.0031
2	1.20	0.0021	1.21	0.0020
3	1.02	0.0016	1.01	0.0015
4	0.87	0.0015	0.86	0.0014
5	0.76	0.0016	0.75	0.0015
6	0.66	0.0017	0.65	0.0016
7	0.59	0.0020	0.58	0.0018
8	0.54	0.0024	0.53	0.0022
9	0.51	0.0032	0.49	0.0029
10	0.50	0.0045	0.48	0.0041

Table 5
Comparison of natural frequencies (Hz) for a clamped–free, glass for epoxy shell with fibre angle 90° of dimensions $R = 18.29\,\text{m}$, $L = 12.2\,\text{m}$, $t = 0.0254\,\text{m}$

Circumferential mode number	Polynomial approach		Ramasamy and Ganesan [2]	
	Frequency (Hz)	Loss factor	Frequency (Hz)	Loss factor
	$t_c/t = 1$	$t_c/t_f = 1$	$t_c/t_f = 1$	$t_c/t_f = 1$
1	2.18	0.0004	2.17	0.0004
2	1.45	0.0005	1.45	0.0004
3	1.05	0.0005	1.04	0.0005
4	0.80	0.0006	0.79	0.0006
5	0.64	0.0009	0.63	0.0008
6	0.53	0.0019	0.52	0.0017
7	0.48	0.0051	0.47	0.0046
8	0.47	0.0114	0.46	0.0103
9	0.51	0.0200	0.50	0.0181
10	0.59	0.0289	0.57	0.0263

6. Conclusions

In the present study, use of the novel polynomial approach to characterise fluid domain has been proposed. Numerical results on the natural frequencies obtained by polynomial approach compared very well with the other procedures. It is felt that polynomial approach will be more elegant and general than Bessel function approach since in the later approach Bessel function values have to be evaluated depending on shell dimensions.

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